

CONSTRUCTION OF EWMA NON-PARAMETRIC CONTROL CHART USING KRUSKAL – WALLIS TEST

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Abstract: Distribution-free control charts can play a useful role in this research, as a parametric model assumption often cannot be adequately verified. For benchmarking reasons, the non-parametric EWMA-chart is included in the comparisons. However, results provide evidence in favour of using distribution-free charts in practice. In this research article we construct the non- parametric exponentially weighted moving average (EWMA) control limits for based on Kruskal-Wallis test with process capability for the data on balance with the presently available control limits.

Keywords: Control chart, Kruskal-Wallis test and Process capability.

1. INTRODUCTION

According to Montgomery (2013), Shewhart-type control charts are particularly useful for statistical analysis because they are easy to construct and interpret and they are effective in detecting large, sustained shifts in the process parameters and outliers, measurement errors, data recording, and/or transmission errors and the like, which are perhaps expected in the early part of a process development. For a thorough account of the control charting literature, Chakraborti et al. (2009), recognising and highlighting the importance of statistical analysis, a more recent comprehensive account of it is given in Jones-Farmer et al. (2014). Many of the available control charts in the literature are parametric control charts, based on an assumption of a particular underlying process distribution, such as the normal. The performance of these charts generally deteriorates when the underlying assumptions are not met. In fact, it is well-known that the Shewhart charts are very sensitive to the normality assumption, in that their false alarm rates are inflated, often significantly, when the underlying distribution deviates from normality. Moreover, it is often not possible to know much about the underlying distribution in the early stages of process development and study, so that a specific distributional assumption (such as normality) cannot be reasonably justified. In these settings, nonparametric control charts can provide a useful. Two major advantages of the nonparametric charts are that one does not have to make an assumption about the form and the shape of the underlying process distribution. In this paper, we compare the performance nonparametric EWMA Shewhart control chart and non-parametric EWMA control chart using process capability with an example.

2. CONCEPTS AND TERMINOLOGIES

a. Upper specification limit (USL)

It is the greatest amount specified by the producer for a process or product to have the acceptable performance.

b. Lower specification limit (LSL)

It is the smallest amount specified by the producer for a process or product to have the acceptable performance.

c. Tolerance level (TL)

It is a statistical interval within which, with some confidence level, a specified proportion of a sampled population falls. It is the difference between USL and LSL, $TL = USL - LSL$.

d. Process capability (C_p)

Process capability compares the output of an in-control process to the specification limits by using capability indices. The comparison is made by forming the ratio of the spread between the process specifications to the spread of the process values, as measured by 6 process standard deviation units. i. e. $C_p = \frac{TL}{6\sigma} = \frac{USL - LSL}{6\sigma}$.

3. METHODS AND MATERIALS

A nonparametric Phase I Shewhart-type control chart was proposed by Jones-Farmer et al. (2014). This chart is based on the well-known Kruskal-Wallis test, Gibbons and Chakraborti (2010). We begin by combining the observations from the m Phase I samples in a single pooled sample of size $N = m \times n$ and ordering the observations from the lowest to the highest. Then ranks (R_{ij} where $i = 1, 2, \dots, m$ and $j = 1, 2, \dots, n$) are assigned to each observation of this pooled sample. The average rank for the i^{th} sample is given by

$$\bar{R}_i \pm n^{-1} \sum_{j=1}^n R_{ij} \text{ for } i = 1, 2, \dots, m$$

and a EWMA control scheme (for individual observations) is easy to implement and interpret. For monitoring the process mean, the EWMA control chart consists of plotting

$$Z_i = \lambda X_i + (1 - \lambda)Z_{i-1}, \quad 0 < \lambda \leq 1.$$

together with upper control limits (UCL) and lower control limits (LCL).

This test is appropriate for use under the following circumstances:

- we have three or more conditions that you want to compare;
- each condition is performed by a different group of participants; i.e. you have an independent-measures design with three or more conditions.
- the data do not meet the requirements for a parametric test. (i.e. use it if the data are not normally distributed; if the variances for the different conditions are markedly different; or if the data are measurements on an ordinal scale).

4. CONSTRUCTION OF NON-PARAMETRIC EWMA CONTROL CHART

We find some depressed people and check that they are all equivalently depressed to begin with.

Table 1: Rating on depression scale

S.No	No exercise	Jogging for 20 minutes	Jogging for 60 minutes
1	23	22	59
2	26	27	66
3	51	39	38
4	49	29	49
5	58	46	56
6	37	48	60
7	29	49	56
8	44	65	62

Then we allocate each person randomly to one of three groups: no exercise; 20 minutes of jogging per day; or 60 minutes of jogging per day. At the end of a month, we ask each participant to rate how depressed they now feel, on a Likert scale that runs from 1 ("totally miserable") through to 100 ("ecstatically happy"). The appropriate test here is the Kruskal-Wallis test. We have three separate groups of participants, each of whom gives us a single score on a rating scale. Ratings are examples of an ordinal scale of measurement, and so the data are not suitable for a parametric test. The Kruskal-Wallis test will tell us if the differences between the groups are so large that they are unlikely to have occurred by chance.

4.1 Construction of EWMA control chart:

The control limits suggested by Roberts (1959) based on 3 – Sigma limits are

$$UCL_{EWMA} = \mu_0 + L\sigma \sqrt{\frac{\lambda}{(2-\lambda)}} = 44.47 + (3*0.84) \sqrt{\frac{0.1}{(2-0.1)}} = 45.05$$

$$\text{Central Line, } CL_{EWMA} = \mu_0 = 44.47$$

$$LCL_{EWMA} = \mu_0 - L\sigma \sqrt{\frac{\lambda}{(2-\lambda)}} = 44.47 - (3*0.84) \sqrt{\frac{0.1}{(2-0.1)}} = 43.89$$

From the result, it is clear that the process is out of control, since the sample numbers 5, 6 and 8 go above the upper control limit and the sample numbers 1, 2, 3 and 4 go below the lower control limit.

4.2 Construction of EWMA non-parametric control chart using Kruskal-Wallis test:

The control limits are

$$UCL_{KW} = \mu_{KW} + L\sigma_{KW} \sqrt{\frac{\lambda}{(2-\lambda)}} = 79.5 + (3*8.38) \sqrt{\frac{0.1}{(2-0.1)}} = 85.27$$

$$\text{Central Line, } CL_{KW} = \mu_{KW} = 79.5$$

$$LCL_{KW} = \mu_{KW} - L\sigma_{KW} \sqrt{\frac{\lambda}{(2-\lambda)}} = 79.5 - (3*8.38) \sqrt{\frac{0.1}{(2-0.1)}} = 73.13$$

From the result, it is clear that the process is out of control, since the sample numbers 1, and 2 go above the upper control limit and the sample numbers 7 and 8 go below the lower control limit.

4.3 Construction of EWMA non-parametric control chart using process capability:

Difference between upper specification and lower specification limits is 23.83, which termed as tolerance level (TL) and choose the process capability (Radhakrishnan and Balamurugan, 2010) is 2.0, it is found that the value of σ_{CP} is 1.99. The control limits of using process capability,

$$UCL_{CP} = \mu_{KW} + L\sigma_{CP} \sqrt{\frac{\lambda}{(2-\lambda)}} = 79.5 + (3*1.99) \sqrt{\frac{0.1}{(2-0.1)}} = 80.86$$

$$\text{Central Line, } CL_{CP} = \mu_{KW} = 79.5$$

$$LCL_{CP} = \mu_{KW} - L\sigma_{CP} \sqrt{\frac{\lambda}{(2-\lambda)}} = 79.5 - (3*1.99) \sqrt{\frac{0.1}{(2-0.1)}} = 78.13$$

From the result, it is clear that the process is out of control, since the sample numbers 1, 2 and 3 go above the upper control limit and the sample numbers 5, 6, 7 and 8 go below the lower control limit.

5. CONCLUSION

We believe that the proposed control chart research has not received enough attention, since the vast majority of the statistical process control literature concerns control charting techniques. Our results make a strong case for using nonparametric chart in practice, particularly with the computing resources available today. In this paper we compared the performances of available nonparametric charts. In summary, the considerably better performance of the EWMA non-parametric control chart using process capability with Kruskal – Wallis test compared with that of the EWMA non-parametric control chart for non-normal data outweighs its slightly better performance.

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